BACK PAPER: TOPOLOGY BMATH2, WINTER 2026

Total time: 3 Hours. **All questions are compulsory**. Each question carries 20 marks. You may use results proved in class without proof, but you need to state the result clearly before using it. If a question specifically asks for the proof of a result covered in class, you must provide a detailed proof. If you wish to use a problem from a homework/assignment, supply its solution too. Without a proper explanation, only partial points/no points will be credited.

(1) Let X be a topological space and $A \subset X$.
  (a) Define the interior of A, which we denote by $\text{int}(A)$.
  (b) Show that A is open if and only if $A = \text{int}(A)$.
  (c) Define the closure of a set A, which we denote by $\bar{A}$.
  (d) Show that A is closed if and only if $\bar{A} = A$.
  (e) In the finite complement topology on $\mathbb{R}$, determine $\bar{A}$, where $A = \{1/n \mid n \in \mathbb{N}\}$.

(2) Let p be a prime. For any $x \in \mathbb{Q} \setminus \{0\}$, $x = p^k \frac{m}{n}, k, m, n \in \mathbb{Z}, p \nmid m, n$. Then we define $v_p(x) := k$,

$$d_p(x, y) := \begin{cases} 0 \text{ if } x = y, \\ p^{-v_p(x-y)} \text{ if } x \neq y \end{cases}.$$

  Show the following:
  (a) $(\mathbb{Q}, d_p)$ is a metric space.
  (b) Any ball in $(\mathbb{Q}, d_p)$ is both open and closed.
  (c) Every point in a ball is a center of the ball.
  (d) The sequence (p^n) converges in $(\mathbb{Q}, d_p)$.
  (e) Does the sequence (p_1^n) converges in $(\mathbb{Q}, d_p)$ where p_1 a prime that is not equal to p?

(3) (a) Show that the interval (a, b) is homeomorphic to the interval $(0, 1)$, where $a < b$.
  (b) Show that $\mathbb{R}$ and $\mathbb{R}^n$ are not homeomorphic if $n > 1$.
  (c) Let $f : Y \to \prod_{\alpha \in \mathcal{A}} X_\alpha$ be a map defined by $f(y) := (f_\alpha(y))_{\alpha \in \mathcal{A}}$, where $f_\alpha : Y \to X_\alpha$ are maps, and $\prod_\alpha X_\alpha$ is equipped with product topology. Show that f is continuous if and only if each f_α is continuous.
  (d) Will the above statement in $3(c)$ be true if $\prod_\alpha X_\alpha$ is equipped with box topology? If yes, provide the proof; otherwise, provide a counterexample with an explanation.
  (e) Let $B[0, 1]$ be the unit closed ball in $\mathbb{R}^n$ Show that the quotient space obtained from $B[0, 1]$ by identifying its boundary $\mathbb{S}^{n-1}$ to a point is homeomorphic to $\mathbb{S}^n$.